CBCS SCHEME

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18MCM/MTR/MAR/IAE11

First Semester M.Tech. Degree Examination, Aug./Sept.2020
Applied Mathematics

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Explain about, (i) Inherent error (ii) Truncation errors (iii) Round off errors (iv) Relative error. **(08 Marks)**

b. Perform 5 iterations of the bisection method to obtain the smallest positive root of the equation $f(x) = x^3 - x - 1 = 0$. **(07 Marks)**

c. Find a real root of the equation $x = e^{-x}$ using the Newton-Raphson method. **(05 Marks)**

OR

2 a. Prove that the terminal velocity of the freely falling body parachutist is $V = \frac{gm}{C}\left[1 - e^{-(C/m)t}\right]$ in the usual notation. **(08 Marks)**

b. The equation $2x = \log_{10} x + 7$ has a root between 3 and 4. Find this root, correct to 3 decimal places, by Regula-Falsi method. **(07 Marks)**

c. Find a real root of the equation, $x^3 - 2x - 5 = 0$, using Secant method. **(05 Marks)**

Module-2

3 a. Using Muller's method, find the root of the equation $f(x) = x^3 - 5x + 1 = 0$. **(10 Marks)**

b. Evaluate $\int_0^1 \frac{dx}{1+x}$, correct to 3 decimal places, with h = 0.5, h = 0.25 and h = 0.125 respectively using Trapezoidal rule. **(10 Marks)**

OR

4 a. Evaluate the integral $I = \int_0^1 \frac{2x\,dx}{1+x^4}$, using the Gauss Legendre's 1-point, 2-point and 3-point quadrature rule. **(10 Marks)**

b. Use Romberg's method to compute $\int_0^1 \frac{dx}{1+x}$ correct to 3 decimal places. **(10 Marks)**

Module-3

5 a. Find the inverse of the coefficient matrix of the system $\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 4 \end{bmatrix}$ by Gauss Jordan method. **(10 Marks)**

b. Find the inverse of the matrix $A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$, using Crout's method. **(10 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

OR

6 a. Solve the system of equations,

$x_1 + 2x_2 + 3x_3 = 5$,

$2x_1 + 8x_2 + 22x_3 = 6$,

$3x_1 + 22x_2 + 82x_3 = -10$

Using the Cholesky method. **(10 Marks)**

b. Determine the inverse of the matrix $\begin{bmatrix} 2 & 1 & 1 & -2 \\ 4 & 0 & 2 & 1 \\ 3 & 2 & 2 & 0 \\ 1 & 3 & 2 & -1 \end{bmatrix}$ using partition method. Hence, solve the system of equations AX= b, where $b = [-10, 8, 7, -5]^T$. **(10 Marks)**

Module-4

7 a. Using the Jacobi method find all the Eigen values and the corresponding Eigen vectors of the matrix, $A = \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$. **(10 Marks)**

b. Transform the matrix $A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 1 \end{bmatrix}$ to tridiagonal form by Given's method, hence find Eigen values. **(10 Marks)**

OR

8 a. Using the Householder's transformation rules reduce the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ into a tridiagonal matrix. **(10 Marks)**

b. Find the largest eigenvalue in modulus and the corresponding Eigen vector of the matrix $A = \begin{bmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{bmatrix}$ using Power method. **(10 Marks)**

Module-5

9 a. Find the columns of $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ are $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Suppose T is linear transformation for $\mathbb{R}^2$ into $\mathbb{R}^3$ such that $T(e_1) = \begin{bmatrix} 5 \\ -7 \\ 2 \end{bmatrix}$ and $T(e_2) = \begin{bmatrix} -3 \\ 8 \\ 0 \end{bmatrix}$ with no additional information. Find a formula for the image of a arbitrary X in $\mathbb{R}^2$. **(10 Marks)**

b. Let W = span{x_1, x_2}, where $x_1 = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$ and $x_2 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$. Construct an orthogonal basis $\{V_1, V_2\}$ for W. **(10 Marks)**

OR

10 a. Find a least-squares solution of the inconsistent system AX=b for $A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}, b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$.

(10 Marks)

b. i) Let $u_1 = \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix}$, $u_2 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$ and $y = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Observe that $\{u_1, u_2\}$ is an orthogonal basis for $W = \text{space}\{u_1, u_2\}$. Write y as the sum of a vector in W and a vector orthogonal to W.

(05 Marks)

ii) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation then there exists a unique matrix A such that T(X)=AX for all x in $\mathbb{R}^n$, in fact, A is the $m \times n$ matrix whose j^{th} column is the T(ej), where ej is the j^{th} column of the identity matrix $\mathbb{R}^n$: $A[T(e_1).....T(e_n)]$.

(05 Marks)

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